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TECHNICAL MEMORANDUM

WSRL-0396-TM

BREAKDOWN OF THE MODIFIED POINT MASS MODEL FOR HIGH ELEVATION TRAJECTORIES

R.L. POPE

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S U M M A R Y

The limits of applicability of the quasi-steady state assumption on which the modified shell trajectory model is based have been investigated. Conclusions have been drawn about the applicability of the modified point mass model to fire prediction for very high gun elevations. Implications for calibration of shell fire control models using the Mach number fitting process are particularly interesting.

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1. INTRODUCTION

Modern computerised methods for producing ordnance firing tables are based on the modified point mass trajectory model for a spin stablised projectile(ref.1). The model includes only the effects of the steady-state gravity-induced trim or yaw of repose. In this way significant contributions from the angular motion of the projectile are included in the trajectory computation, but the penalties arising from the very small integration step size necessary for accurate representation of the transient angular motion are avoided. The modified point mass model is valid for a dynamically stable missile and slowly varying yaw of repose. Murphy(ref.2) has shown that for a sufficiently high angle of fire the quasi-steady state assumption breaks down near apogee, where conditions change most rapidly. When this occurs precession or the slow mode of the epicyclic motion is excited and transient motion persists on the downleg. As a result trajectory predictions with the modified point mass model show significant range errors for some high angle trajectories.

Murphy suggested in reference 2 that a further modification could be incorporated in the modified point mass model to correct this problem. The idea has not been followed up, principally because the gun elevations which produce such behaviour are not used a great deal in practice. However, the break down of the quasi-steady assumption does have implications for the calibration of the modified point mass trajectory model for shell fire control purposes. The modern Mach number fitting technique for calibration of shell fire control models(ref.3), using data from Range and Accuracy trials, relies on the overall accuracy of the trajectory model. Severe distortions may arise in the calibration process, and in using the model for prediction purposes if there are significant errors in trajectory predictions for high angle fire.

This brief report summarises the results in reference 2 and looks at the magnitude of the transient motion induced by the rapidly changing conditions near apogee and the relation between that motion and gun elevation. Results are given for a variety of shells of interest to the Australian Army. Finally some conclusions are drawn about the relevance of the effect to practical trajectory calculation, with particular emphasis on model calibration and Range and Accuracy trials.

2. THEORY

Two right handed orthogonal sets of axes are used in describing projectile motion. The earth fixed axes have the x_e-axis downrange, z_e-axis vertically downwards and y_e-axis to the right. The aeroballistic axes have the $\hat{x}$-axis along the axis of symmetry of the projectile, the z-axis in the plane of the trajectory pointing downwards and the y-axis horizontal to the right. The complex angle of attack, defined in the aeroballistic axes system is

$$\hat{\xi} = (\hat{v} + i\hat{w})/V,$$

wher $\hat{v}$, $\hat{w}$ are $\hat{y}$ and $\hat{z}$ velocity components, so that the real part of $\hat{\xi}$ is in a horizontal plane and the imaginary part is in a vertical plane. A detailed derivation of the gravity-induced yaw of repose for a spin stabilised projectile is given in reference 1. A very good approximation for the yaw of repose(ref.2) is

$$\delta_g = pI_x g_z/QSdVC_{m_\alpha} \qquad (1)$$

where p = roll rate,

I_x = axial moment of inertia,

g_z = gravity component along z-axis,

Q = dynamic pressure,

S = projectile cross-sectional area,

d = projectile calibre,

V = relative air velocity of projectile, and

C_{m_α} = overturning moment coefficient.

If the yaw of repose is rapidly varying near apogee the total yaw can be represented by

$$\xi_g = -\delta_g + \delta_{ga} K_G (\delta_{ga}, \phi_2) e^{i\phi_2} \tag{2}$$

where δ_{ga} is the value of δ_g at apogee, and

ϕ_2 is the phase angle of the slow mode of the epicyclic motion and damping of the slow mode is ignored for simplicity. The phase angle can be derived by integrating the equation

$$\dot{\phi}_2 = (pI_x/I_y) [1 - (1-s_g{}^{-1})^{\frac{1}{2}}] \tag{3}$$

where $s_g = (pI_x)^2/4QSd\, I_y C_{m_\alpha}$ is the gyroscopic stability factor. The amplitude function $K_G(\delta_{ga},\phi_2)$ is defined by the relations

$$K_G = 0, \qquad \phi_2 \leq -2\pi, \text{ upleg}$$

$$K_G = \delta_{ga} \int_{-2\pi}^{\phi_2} e^{-i\phi_2} f(\theta_T)\, d\phi_2, \quad -2\pi < \phi_2 < 2\pi, \text{ near apogee}$$

$$K_G = i\, K_{2G} (\delta_{ga}) \qquad \phi_2 \geq 2\pi, \text{ downleg} \tag{4}$$

where

$$K_{2G} = -\delta_{ga} \int_{-2\pi}^{2\pi} f(\theta_T) \sin\phi_2{}'\, d\,\phi_2{}'$$

and the function $f(\theta_T)$ is defined by the relations

$$f(\theta_T) = 4 \cos^7 \theta_T \sin \theta_T \tag{5}$$

$$3\delta_{ga} \phi_2 = -\tan \theta_T (3 + \tan^2 \theta_T) \tag{6}$$

since θ_T is the angle between the trajectory and its projection onto the horizontal plane. The zero for the phase angle ϕ_2 is chosen to coincide with apogee, when θ_T=0, so that $\phi_2 < 0$, relates to the upleg of the trajectory and $\phi_2 > 0$ to the downleg. Then the total yaw on the downleg of the trajectory is

$$\xi_G = -\delta_g + i \, \delta_{ga} K_{2G} (\delta_{ga}) e^{i\phi_2} . \tag{7}$$

The significant effect of the increased yaw on the trajectory is to increase the yaw drag. The overall drift will not be substantially changed, because only averaged results affect the impact point and the averaged effects of the change in lift will be small. The total drag coefficient is defined as

$$C_D = C_{D_o} + C_{D_{\alpha^2}} \xi_G^2 . \tag{8}$$

On the upleg of the trajectory K_G=0, so that the modified point mass model is accurate. Near apogee, the projectile velocity is low and small changes in drag do not affect the trajectory significantly. However, on the downleg, the effects can be significant. The object of this paper is to establish the conditions under which the effect may be significant and to look at methods for dealing with the resulting problems in using the modified point mass model for fire prediction.

3. NUMERICAL RESULTS

The first step in evaluating the significance of the effect is to determine the amplitude excited in the slow mode of the motion. Figure 1 shows how the function K_{2G} (δ_{ga}) varies with yaw of repose at apogee and the resultant amplitude of the epicyclic motion which is excited. The initial amplitude of the epicyclic motion has a negative maximum of about 0.75° for a yaw of repose at apogee of about 6°. The amplitude then changes sign and the magnitude does not reach 1° until yaw of repose is almost 10°, after that it increases rapidly. Figure 2 shows a typical example of the motion excited. It depicts the incidence history for an M1 shell launched at an elevation of 70°. The slow mode motion on the downleg is typical of this sort of trajectory.

In order to evaluate the consequences of these results let us look at how yaw of repose varies with gun elevation for a variety of shells. Figure 3 shows yaw of repose for two 105 mm projectiles and two 155 mm projectiles. Results are plotted for muzzle velocities equivalent to maximum charge for each projectile. The yaw of repose for which the induced epicyclic motion has zero amplitude, 8.5°, is achieved for elevations between 63° and 67°. For a gun elevation of 70°, yaw of repose at apogee varies from 14.5° for the M1 shell

to over 30° for the L15. Therefore some shells will exhibit significant excitation of the slow mode of their epicyclic motion for an elevation of 65°, and all shells can be expected to show the effect for an elevation of 70°. Figure 4 shows the range error of the modified point mass model for the M1 shell, which has the slowest development of the phenomenon. The yaw of repose of the M1 shell at apogee is 8.5° for a gun elevation of 66.6°. Above this elevation it is clear from figure 4 that the range error in modified point mass model predictions increases very rapidly. We can therefore regard the elevation at which yaw of repose at apogee is 8.5° as a critical value for the modified point mass model, since the accuracy of model predictions deteriorates rapidly from that point onwards. Table 1 shows the elevations at which this critical value is reached for each of the shells represented in figure 2. Firing tables generally give data for elevations up to 70° and consequently model calibration uses data from Range and Accuracy firings which contains significant components for elevations around 70°. Therefore this effect will produce significant distortion of the calibration and fire prediction process unless handled carefully.

4. CONCLUSIONS

It is clear that for most shells the flaw in the modified point mass model which has been discussed will not have any significant effect for elevations below 65°. Since the need to use artillery at higher elevations that this is extremely rare, it appears at first glance that the effect can be quite safely ignored. However, problems do arise in the production of firing tables. In general, firing tables are produced for elevations up to 70°, and therefore the Range and Accuracy trials on which the tables are based often contain up to 20% of data points from firings at elevations between 65° and 70°, and sometimes even more. Because of the rapidly deteriorating performance of the modified point mass trajectory model at these elevations, the calibration process has to cope with significant errors from this part of the data which will produce corresponding distortions of the fitting process at low to moderate elevations. The effects will be particularly severe for the yaw drag calibration factor, QFAC, in the Mach number fitting method(ref.3) because of the global nature of the process.

The alternative suggested in reference 3 of a revision of the modified point mass scale would probably improve the trajectory representation sufficiently to avoid distortion and bias effects. However, since the firing tables are used so rarely at these high elevations it is possible to restrict the ballistic data gathering trials and the follow up calibration process to elevations for which the yaw of repose at apogee is below the critical value of 8.5°. This would form an additional constraint on the design method for Range and Accuracy Trials, described in reference 4.

TABLE 1. CRITCAL GUN ELEVATIONS FOR $\delta_{ga} = 8.5°$

Projectile	Calibre (mm)	MV (m/s)	Elevation (°)
M1	105	464.8	66.6
L31	105	708.0	65.2
L15	155	826.2	63.5
M483A1	155	643.0	64.7

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No.	Author	Title
1	Lieske, R.F. and Reiter, M.L.	"Equations of Motion for a Modified Point Mass Trajectory". TRL Rpt 1314, March 1966
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3	Fitch, P.R.	"A Method of Obtaining Ballistic Calibration Coefficients from Artillery Shell Trials". RARDE Branch Memo 50/78 (GR4), June 1978
4	Nichols, D.A.	"A Summary of Mach Number Fitting and Range and Accuracy Trials Design Techniques". Vol.1 Report, Vol.2 A Worked Example, SCICON Report, June 1984

NOTATION

C_D	total drag coefficient
C_{D_o}	zero yaw drag coefficient
$C_{D_{\alpha^2}}$	yaw drag coefficient
C_{m_α}	overturning moment coefficient
d	projectile calibre
g_z	component of gravity along z axis
I_x	axial moment of inertia
I_y	transverse moment of inertia
$K_G(\delta_{ga},\phi_2)$	amplitude of slow mode motion induced by gravitational effects
$K_{2G}(\delta_{ga})$	amplitude of slow mode motion on downleg
p	projectile spin rate
Q	dynamic pressure
S	projectile cross sectional area
s_g	gyroscopic stability factor
V	projectile velocity
$(\hat{u},\hat{v},\hat{w})$	velocity components in aeroballistic axes
(x_e,y_e,z_e)	inertial earth axes system
$(\hat{x},\hat{y},\hat{z})$	aeroballistic body axes system
δ_g	gravity induced quasi-steady yaw of repose
δ_{ga}	yaw of repose at apogee
θ_T	instantaneous elevation of trajectory
ξ	total imaginary yaw
ξ_g	quasi-steady gravity induced yaw of repose
ξ_G	unsteady gravity induced yaw
ϕ_2	phase angle of slow mode of epicyclic motion

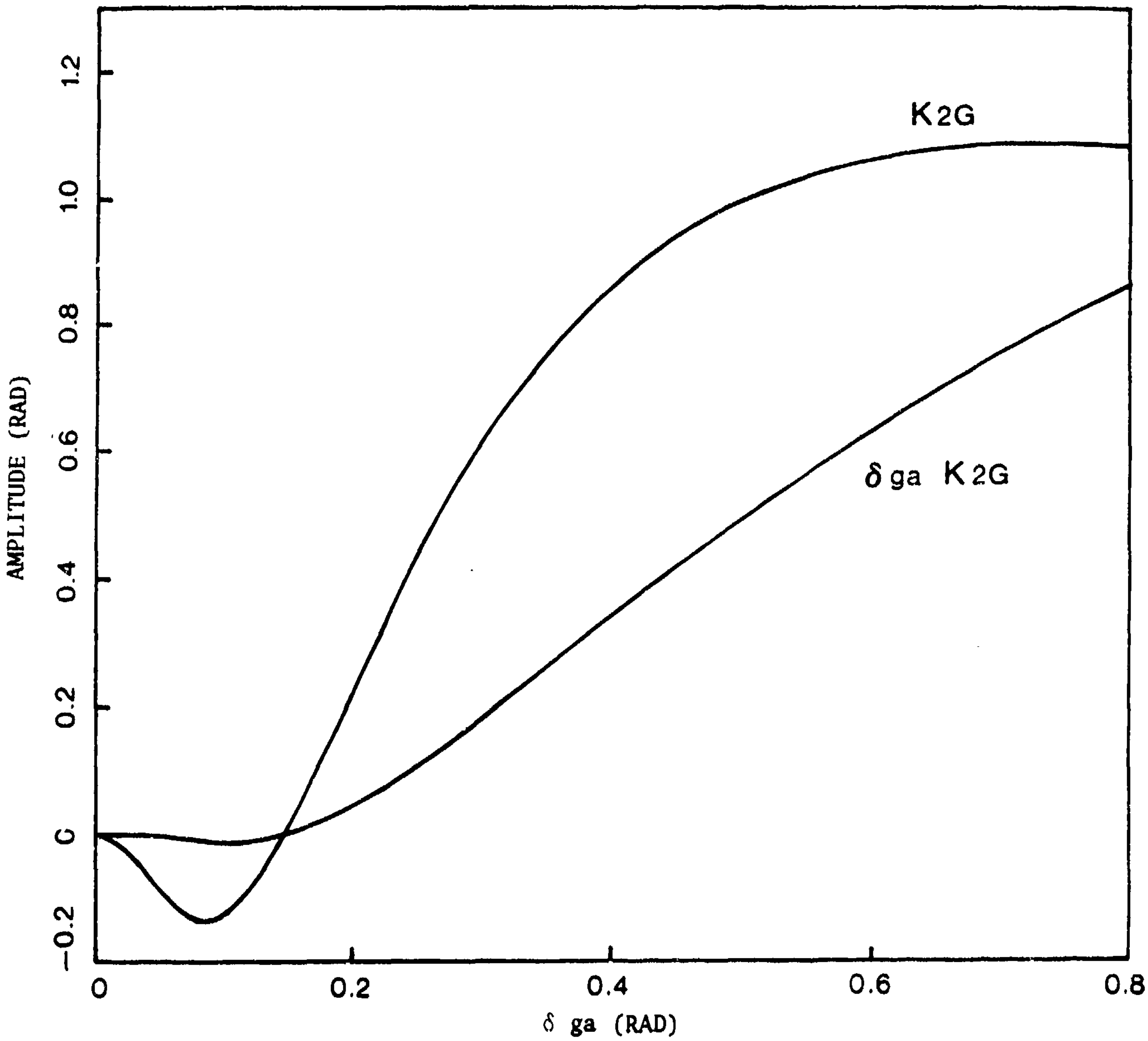

Figure 1. Amplitude of epicyclic motion

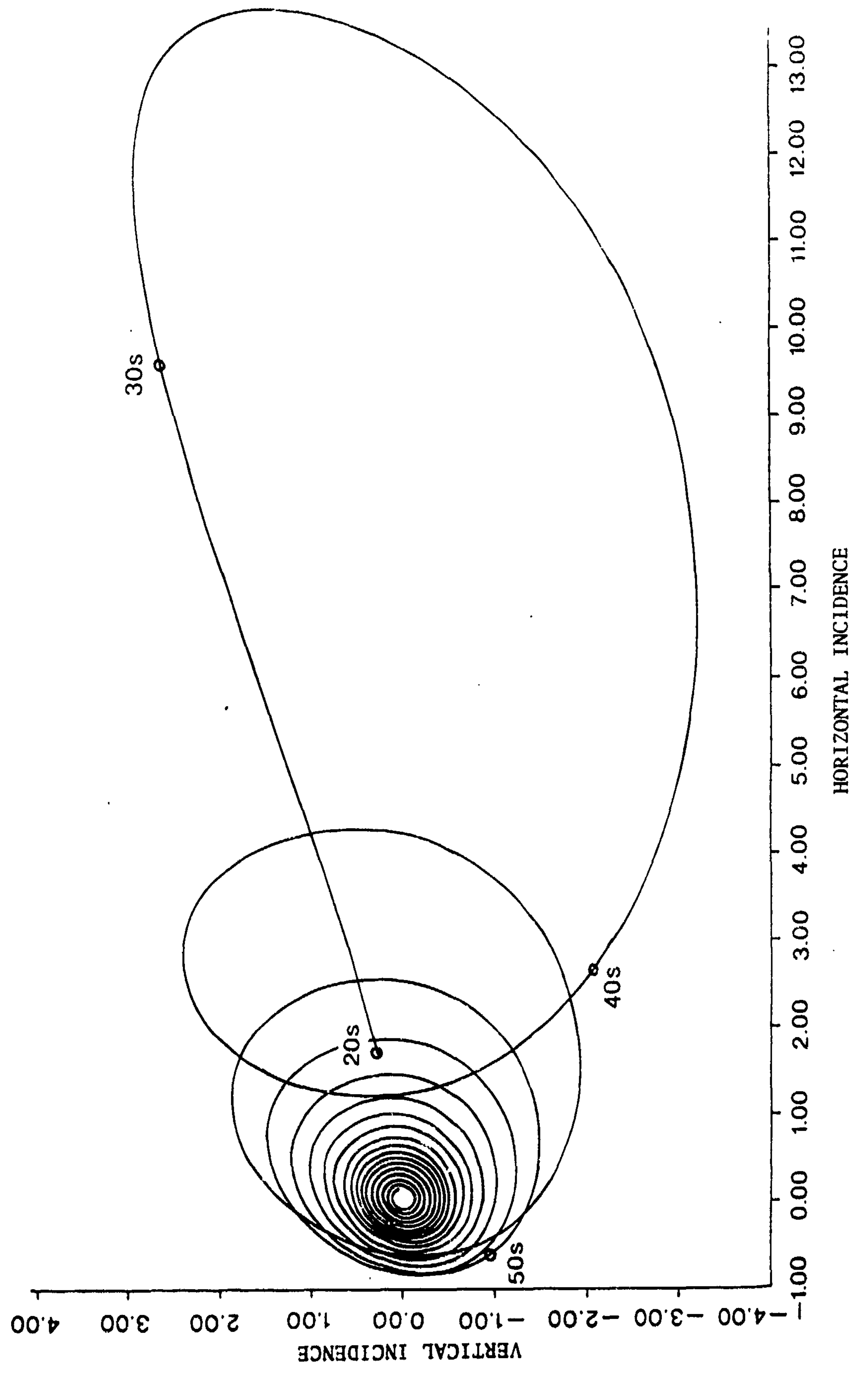

Figure 2. Typical incidence behaviour QE=70°

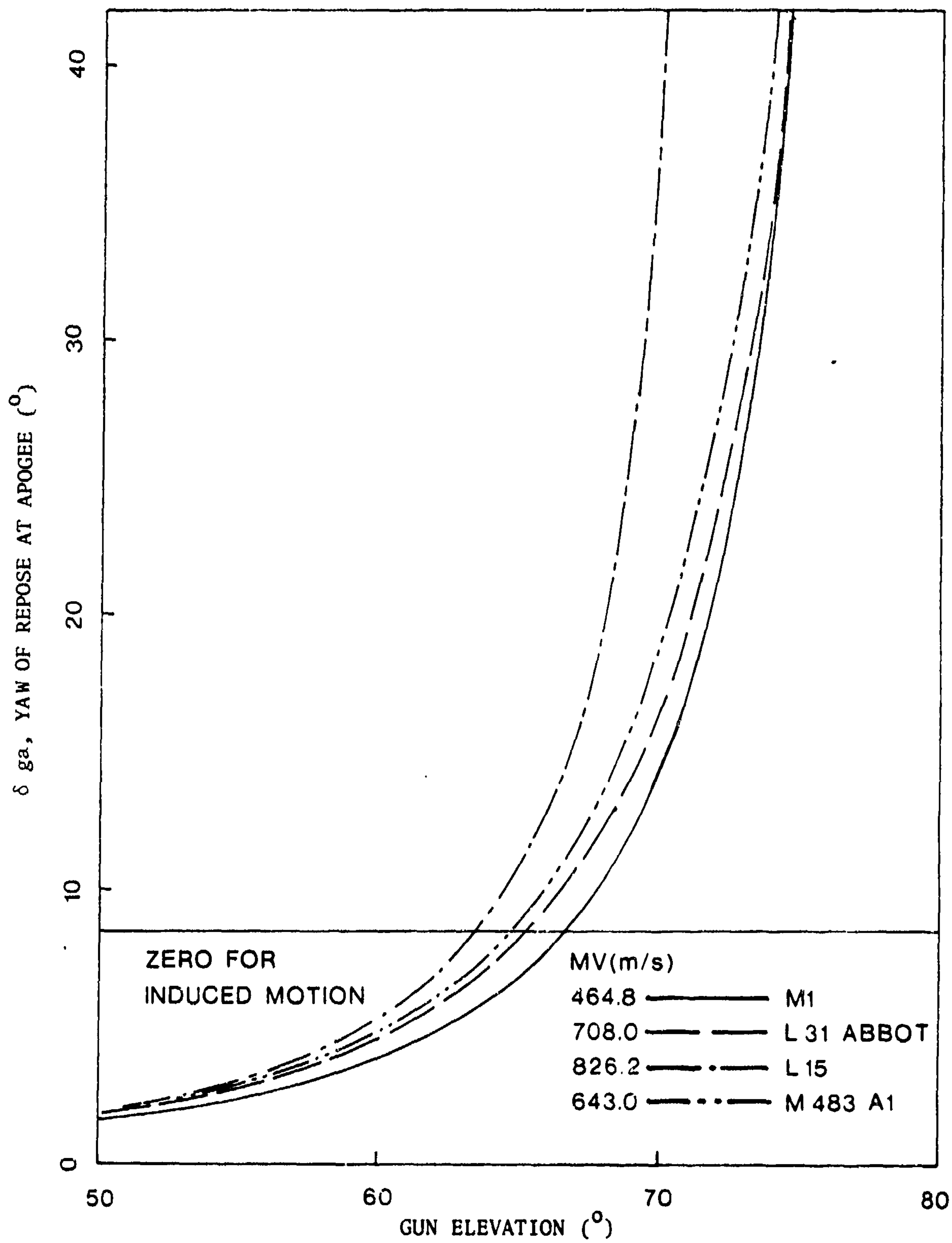

Figure 3. Yaw of repose at apogee for a variety of shells

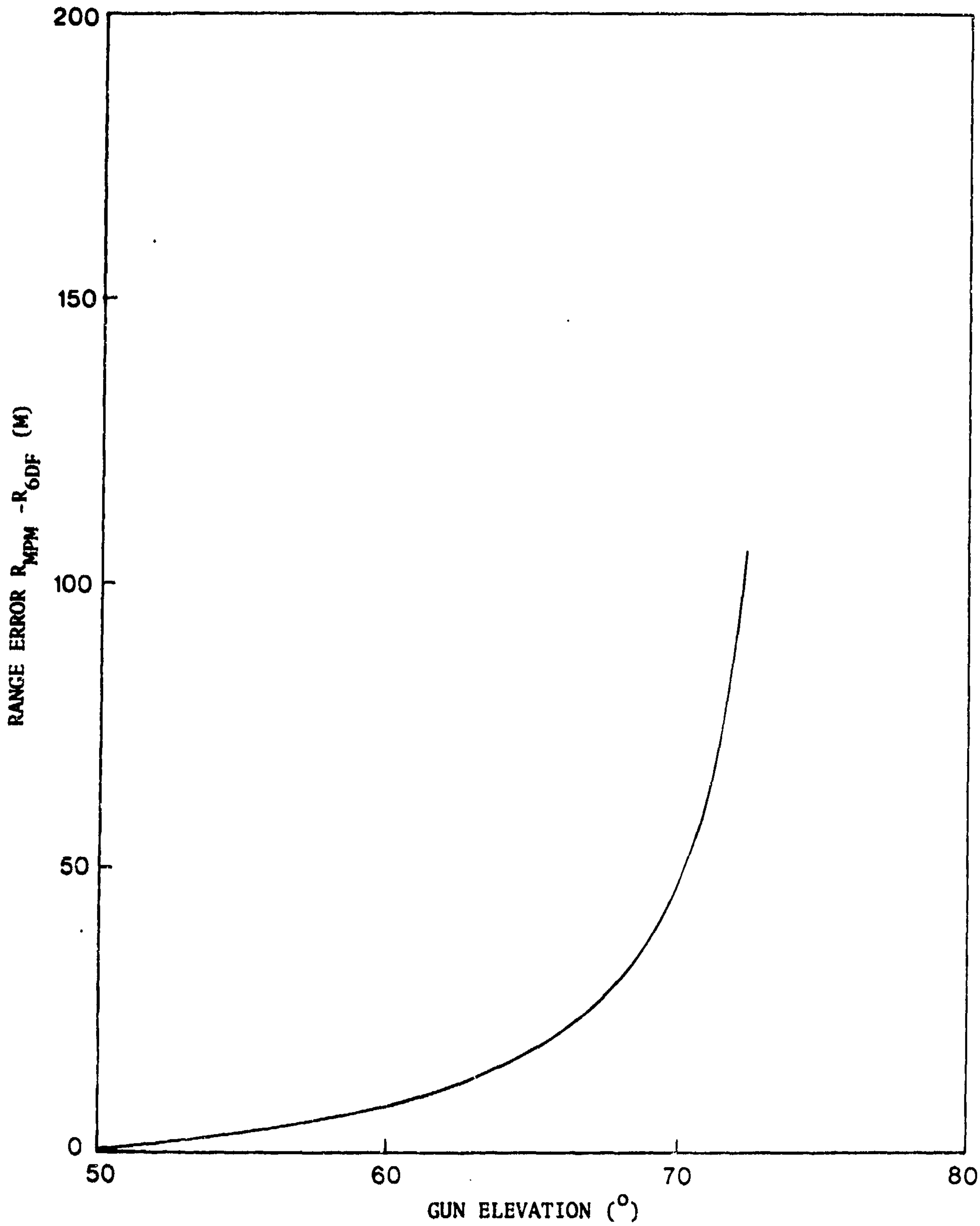

Figure 4. Range error of modified point mass model for M1 shell

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